

Minnesota State High School  
Mathematics League  
Individual Event

3B 1991-92

**Instructions:** The first question should require very little time or computation. It is worth 1 point. Each of the other questions is worth 2 points. Put your answer to each question on the indicated blank on this page.

- \_\_\_\_\_ 1. Given a square ABCD with area  $a^2$  (Figure 1), a line  $\ell$  is drawn through C, parallel to diagonal BD, and sides AB and AD are extended to meet  $\ell$  in E and F. What is the area of trapezoid BDFE?
- \_\_\_\_\_ 2. A diameter of a circle of radius 3, centered at C, is extended in each direction to points P and R so that PC = RC = 6. From P and R, tangents are drawn, intersecting at Q and S (Figure 2). Find the perimeter of quadrilateral PQRS.
- \_\_\_\_\_ 3. The quadrilateral ABCD has right angles at A and B, and its diagonals intersect at right angles at E; AE = 6 and BE = 2 (Figure 3). What is the area of  $\triangle DEC$ ?
- \_\_\_\_\_ 4. [ARML, 1991] The bases of an isosceles trapezoid are 20 and 30, and its altitude is 12. Using each leg of the trapezoid as a diameter, semi-circles are drawn exterior to the trapezoid (Figure 4). If the midpoints of the arcs are P and Q, find the length of PQ.

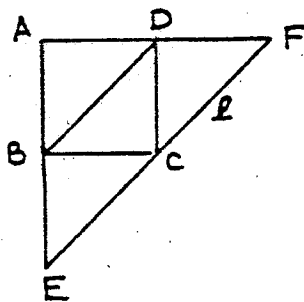


Figure 1

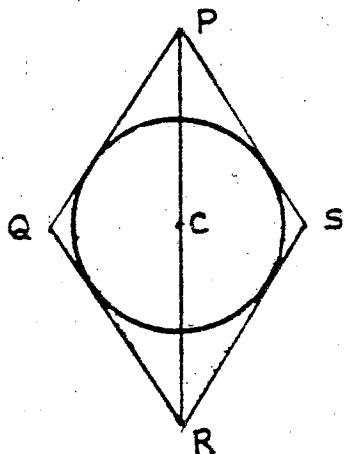


Figure 2

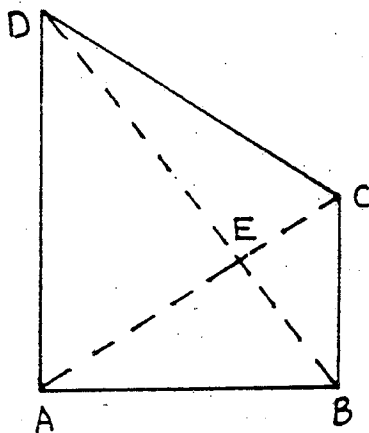


Figure 3

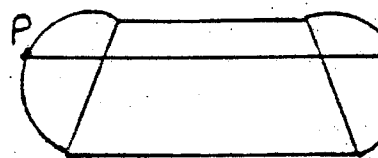
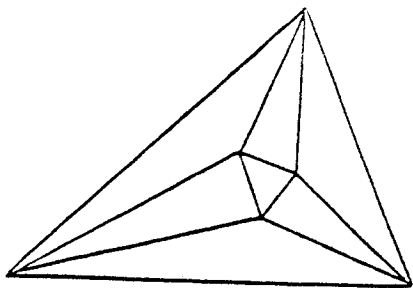


Figure 4

Name \_\_\_\_\_

School \_\_\_\_\_



Minnesota State High School  
Mathematics League  
Individual Event

3B 1992-93

Instructions: The first question should require very little time or computation. It is worth 1 point. Each of the other questions is worth 2 points. Put your answer to each question on the indicated blank on this page.

- \_\_\_\_\_ 1. In the trapezoid ABCD (Figure 1),  $m\angle A = 57^\circ$  and  $m\angle C = 132^\circ$ . What is  $m\angle D$ ?
- \_\_\_\_\_ 2. The mid-points of the sides of a regular pentagon  $A_1A_2A_3A_4A_5$  are joined together to form a second regular pentagon  $B_1B_2B_3B_4B_5$ . What is the measure of  $\angle A_1B_1B_5$  in degrees?
- \_\_\_\_\_ 3. The vertices of a regular octagon of side length 1 are labeled in Figure 3. Midpoints of  $A_2A_3$ ,  $A_3A_4$ ,  $A_5A_6$ , and  $A_8A_1$  are R, S, T, and U respectively. They are connected to form a trapezoid RSTU. How long is the side ST?
- \_\_\_\_\_ 4. A square is inscribed in a regular hexagon of edge length 2 in such a way as to have two edges parallel to two edges of the hexagon (Figure 4). Find in rationalized form the length of an edge of the square.

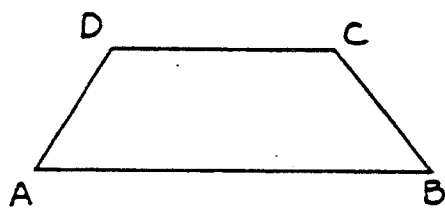


Figure 1

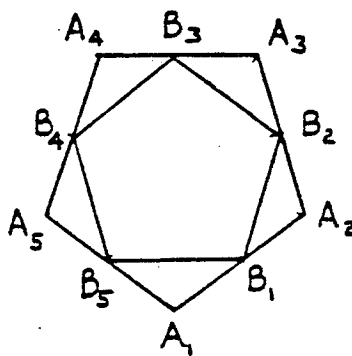


Figure 2

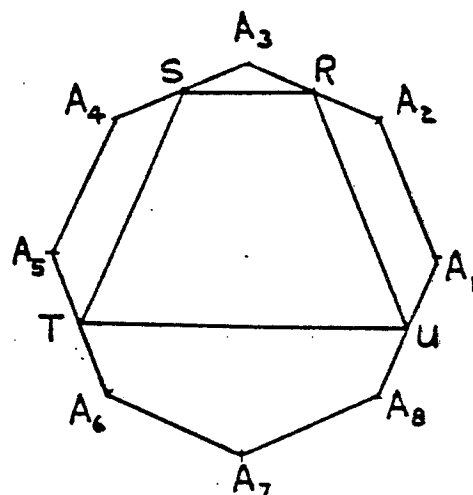


Figure 3

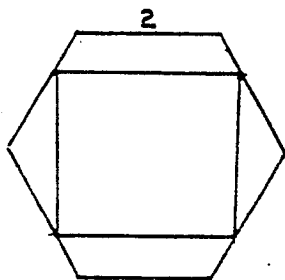
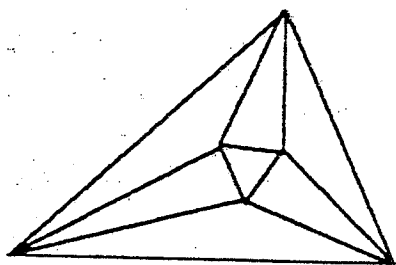


Figure 4

Name \_\_\_\_\_

School \_\_\_\_\_



# Minnesota State High School Mathematics League

## Individual Event

### 1993-94 Event 3B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- \_\_\_\_\_ 1. In the regular octagon pictured in Figure 1, the length of the diagonal  $GC$  is 8. How long is segment  $HB$ ?
- \_\_\_\_\_ 2. A regular octagon has been cut out of paper, and the vertices are being folded in along dotted lines joining the mid-points of the sides. Two such folds have already been made in the case shown in Figure 2. What is the measure in degrees of  $\angle FXE$ ?
- \_\_\_\_\_ 3. The parallelogram in Figure 3 has sides of lengths 7 and 3, and a short diagonal of length 6. How long is the long diagonal?
- \_\_\_\_\_ 4. In Meet 1 this year, teams were asked how many values of  $n$  would give a regular  $n$ -sided polygon having interior angles whose measures were integral. The sequence of values that work, in order of increasing  $n$ , is 3, 4, 5, 6, 8, 9, 10, ... What is the 19th term of this sequence?

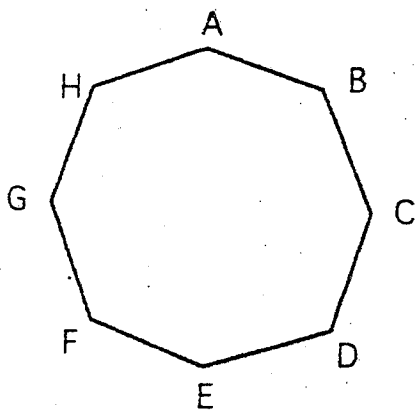


Figure 1

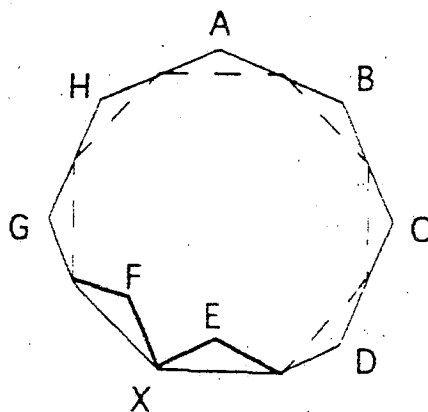


Figure 2

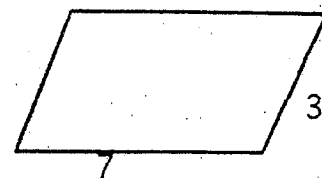
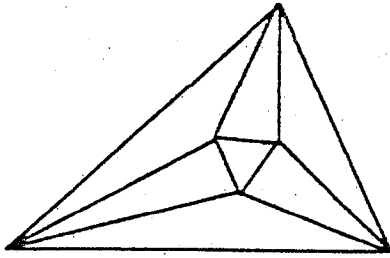


Figure 3

Name \_\_\_\_\_

Team \_\_\_\_\_



# Minnesota State High School Mathematics League Individual Event

## 1994-95 Event 3B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- \_\_\_\_\_ 1. A square of side length 1 is to be covered by a piece of paper cut in the shape of an equilateral triangle as in Figure 1. What is the length of the side of the smallest equilateral triangle that will do the job?
- \_\_\_\_\_ 2. A square of side length 1 is to be covered by a piece of paper cut in the shape of a circle. What is the radius of the smallest circle that will do the job?
- \_\_\_\_\_ 3. Two poles, both perpendicular to the ground, have heights  $a$  and  $b$ , and are separated by a distance of  $c$  at their bases. Each pole is supported by a guy wire that runs from the top of the pole to the base of the other pole (Figure 3). It is a fact that these wires cross at a height  $h$  that depends only on  $a$  and  $b$ , and not at all on  $c$ . Express  $h$  as a function of  $a$  and  $b$ .
- \_\_\_\_\_ 4. Each vertex of a unit square is joined to one of the points of trisection on the opposite side as in Figure 4. Four congruent right triangles, shaded in the figure, are thus formed in the four corners. What is the length of the short leg of these triangles?

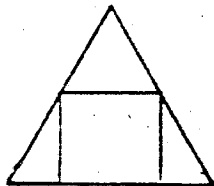


Figure 1

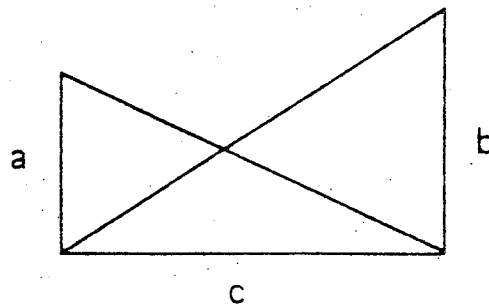


Figure 3

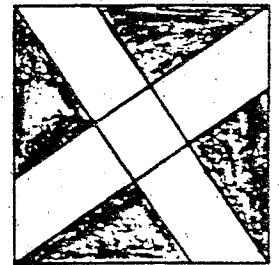
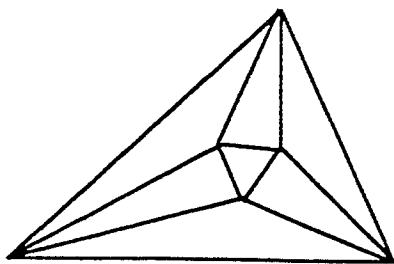


Figure 4

\_\_\_\_\_  
Name

\_\_\_\_\_  
Team



# Minnesota State High School Mathematics League

## Individual Event

### 1995-96 Event 3B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- \_\_\_\_\_ 1. Isosceles trapezoid RSTU (Figure 1) has base angles of  $65^\circ$ . UV is drawn parallel to TS and TW is drawn parallel to UR. What is the measure of  $\angle TUV$ ?
  
- \_\_\_\_\_ 2. Referring again to the trapezoid described in Problem 1 and pictured in Figure 1, suppose isosceles trapezoid RSTU has bases of 12 and 8 and an area of 50. What is the area of trapezoid TUVW?
  
- \_\_\_\_\_ 3. Suppose the construction described in Problem 1 was carried out for an arbitrary isosceles trapezoid with bases of length  $B = \overline{RS}$  and  $b = \overline{UT}$ . What inequality involving  $B$  and  $b$  must be satisfied if trapezoid TUVW is non degenerate; that is,  $\overline{VW} > 0$ ?
  
- \_\_\_\_\_ 4. If the construction described in Problem 1 can be carried out to form a non degenerate trapezoid TUVW, what is the ratio, in terms of  $B$  and  $b$ , of the area of RSTU to the area of TUVW?

Note: The area of a trapezoid with bases of  $b$  and  $B$  is  $A = \frac{1}{2}(b+B)h$  where  $h$  is the perpendicular distance between the bases.

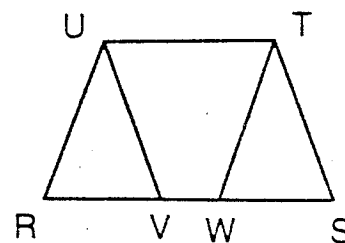
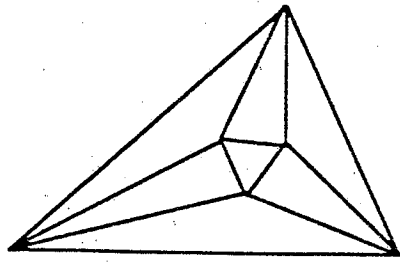


Figure 1

\_\_\_\_\_  
Name

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Team



# Minnesota State High School Mathematics League

Individual Event

## 1996-97 Event 3B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

\_\_\_\_\_ 1. The ancients hoped to construct a square having the same area as a given circle. If the circle has radius 1, exactly what would the side of the square have to be?

2. An isosceles trapezoid has bases of 8 and 14. Express the height  $h$  as an algebraic function of the length  $s$  of the sides.

\_\_\_\_\_

3. In rectangle  $ABCD$  with side lengths  $BC=a$  and  $AB=b$ , (Figure 3), perpendiculars are dropped from  $B$  and  $D$  to  $E$  and  $F$  on the diagonal  $AC$ . What must be the ratio of  $a$  to  $b$  (i.e.  $\frac{a}{b}$ ) so that  $E$  and  $F$  partition  $AC$  into three equal segments?

\_\_\_\_\_

4.  $E$  marks the midpoint of side  $BC$  of a square piece of paper  $ABCD$  (Figure 4a). The paper is then folded so that  $D$  coincides with  $E$ , making  $AD$  cross the top edge at  $T$  (Figure 4b). Find the length of  $TB$  in terms of the length  $s$  of the side of the square.

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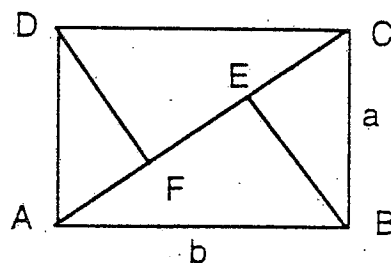


Figure 3

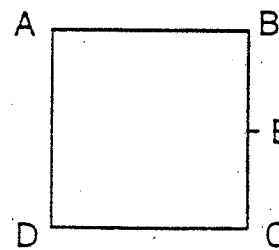


Figure 4a

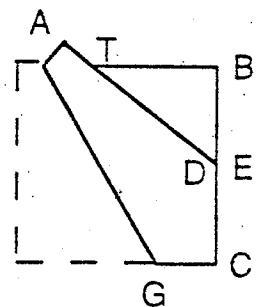
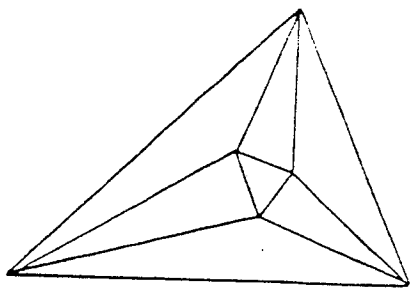


Figure 4b

Name \_\_\_\_\_ Team \_\_\_\_\_

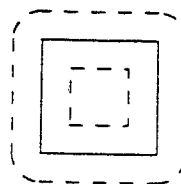


# Minnesota State High School Mathematics League Individual Event

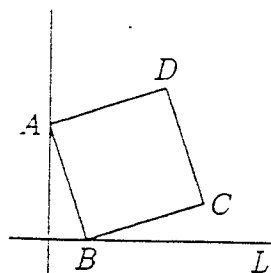
## 1997-98 Event 3B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

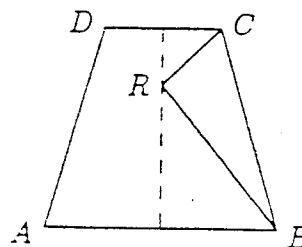
- \_\_\_\_\_ 1. Let  $R$  denote the set of points in the plane that are within one unit of the perimeter of a square of side length 4. What is the area of  $R$ ?



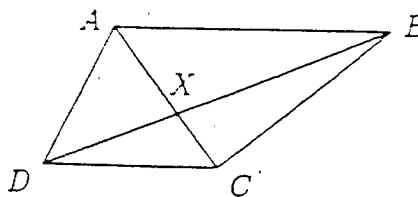
- \_\_\_\_\_ 2. A square  $ABCD$  of side length 1 rests on a horizontal line  $L$  and is tilted against a vertical wall, as shown in the figure at the right. Express the height of the corner  $D$  above  $L$  as a function of the distance  $x$  between the wall and corner  $B$ , for  $0 \leq x < \sqrt{2}/2$ .



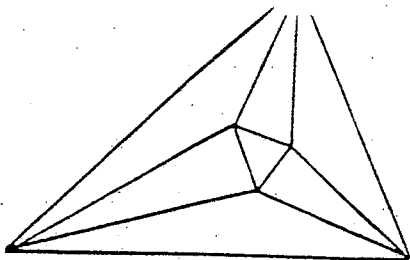
- \_\_\_\_\_ 3. An isosceles trapezoid  $ABCD$  has bases  $AB$  of length 10 and  $CD$  of length 8, and they are 12 units apart. A point  $R$ , closer to  $CD$  than  $AB$ , is on the axis of symmetry of the trapezoid so that  $\angle CRB = 90^\circ$ . Find the distance from  $R$  to  $CD$ .



- \_\_\_\_\_ 4. The diagonals  $AC$  and  $BD$  of trapezoid  $ABCD$  intersect at  $X$ , with  $AB$  parallel to  $CD$ . Find the length of  $CD$  given that  $AB = 4$ ,  $CX = 1$ , and  $AC = CD$ . (Give the answer to five places after the decimal point.)



Name \_\_\_\_\_  
School \_\_\_\_\_



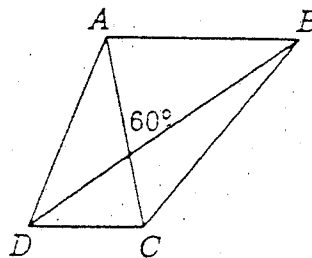
Minnesota State High School  
Mathematics League  
*Individual Event*

**1998-99 Event 3B**

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- \_\_\_\_\_ 1. The measures of four angles of a convex quadrilateral (all angles  $< 180^\circ$ ) are  $x$ ,  $2x$ ,  $3x$ , and  $4x$ . What is the value of  $x$  in degrees?
- \_\_\_\_\_ 2. The measures of three of the four angles of a convex quadrilateral (all angles  $< 180^\circ$ ) are  $x$ ,  $2x$ , and  $3x$ . Determine the range of values in degrees that  $x$  can take on.
- \_\_\_\_\_

- \_\_\_\_\_ 3. The diagonals of trapezoid  $ABCD$  intersect in an angle of  $60^\circ$  (see figure). Also,  $AC = 7$  and  $BD = 12$ . Find the area of  $ABCD$ . Give your answer rounded correct to three places after the decimal.



- \_\_\_\_\_ 4. Find the area of a rhombus with side length 10 and whose diagonals differ by 4. Express your answer as an integer.

Name \_\_\_\_\_  
School \_\_\_\_\_